# Low-Loss Traveling-Wave Parametric Amplifier Without Dispersion Engineering

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# Phase Matching in Josephson Traveling-Wave Parametric amplifiers (TWPA)

- Four wave mixing (FWM):

$$L(I) = L_0\left[1 + \frac{I^2}{2I_c^2}\right]$$

- Energy conservation:

$$2\omega_p = \omega_s + \omega_i$$

- Chromatic dispersion:

$$k = \omega/v_p$$

$$\Delta k = k_s + k_i - 2k_p$$

- Self phase modulation:

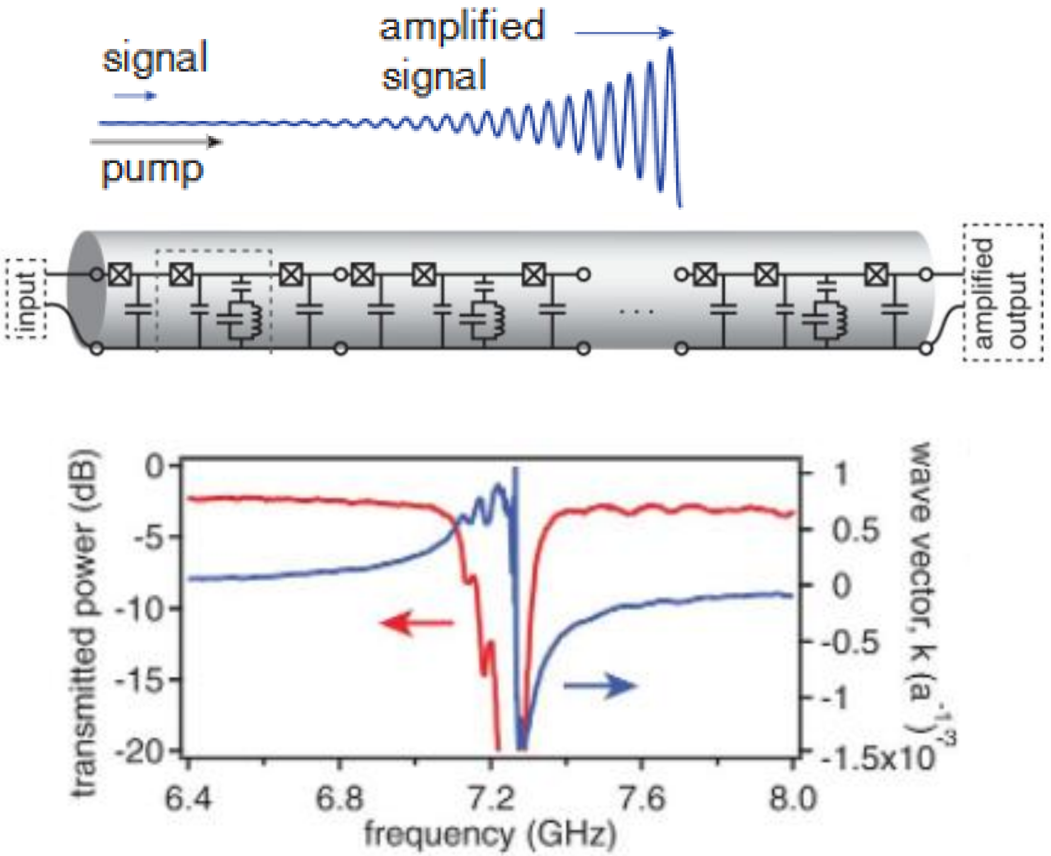
$$\alpha_m \propto \gamma_{JJ}\left|A_{p0}\right|^2 \qquad ,\gamma_{JJ} \text{ Kerr coeff}$$

- Phase matching:

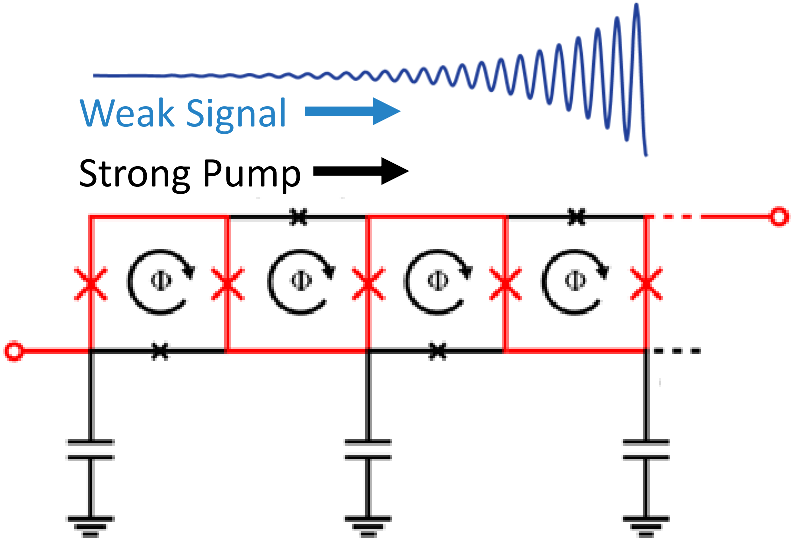
$$\kappa \coloneqq \Delta k + \alpha_s + \alpha_i - 2\alpha_p = 0$$

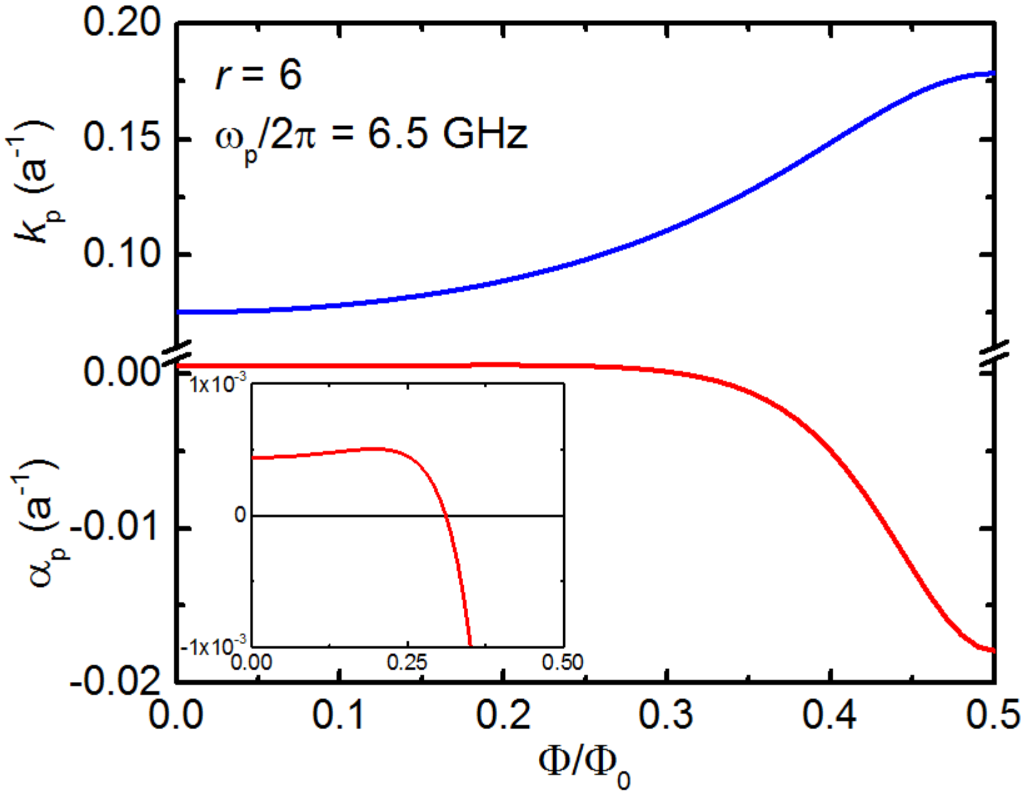
- Exponential gain with phase matching:

$$G_s = |e^{gz}/2|^2$$



# Phase Matching without dispersion engineering through Kerr coefficient



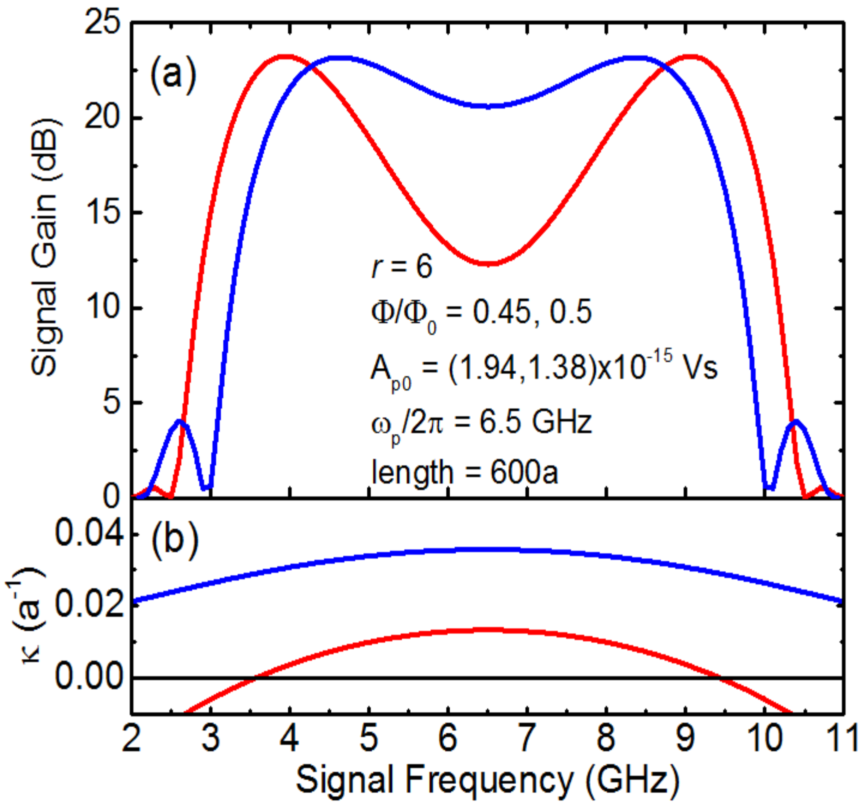

- Cell size $a$
- Small junctions critical current $I_{js}$ and capacity $C_{js}$
- Capacity to ground per unit cell $C_{gnd}$
- Capacity/critical current ratio $r$

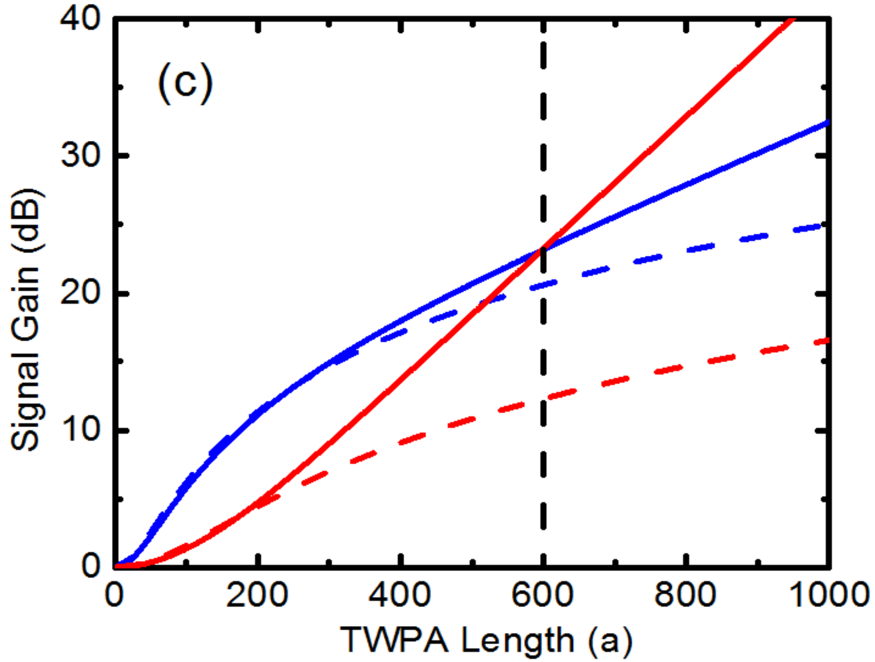
$$\frac{a^2}{L}\left[\frac{r}{2}+2\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\right]\frac{\partial^2\varphi}{\partial z^2}+a^2C_{js}\left(\frac{r}{2}+2\right)\frac{\partial^4\varphi}{\partial t^2\partial z^2}-C_{gnd}\frac{\partial^2\varphi}{\partial t^2}=\gamma\frac{\partial}{\partial z}\left[\left(\frac{\partial\varphi}{\partial z}\right)^3\right]$$

- Kerr coeff: $\gamma=\frac{a^4}{3\varphi_0^2L}\left(\frac{r}{16}+\cos\left(\frac{2\pi\Phi}{\Phi_0}\right)\right)$

- Pump amplitude: $A(z)=A_0e^{-i(k_p+\alpha_p)z}$
- Pump self shift: $\alpha_p\propto\gamma|A_{p0}|^2$

# Phase Matching without dispersion engineering through Kerr coefficient




$$G_s = \left|\cosh(gz) - \frac{i\kappa}{2g}\sinh(gz)\right|^2$$

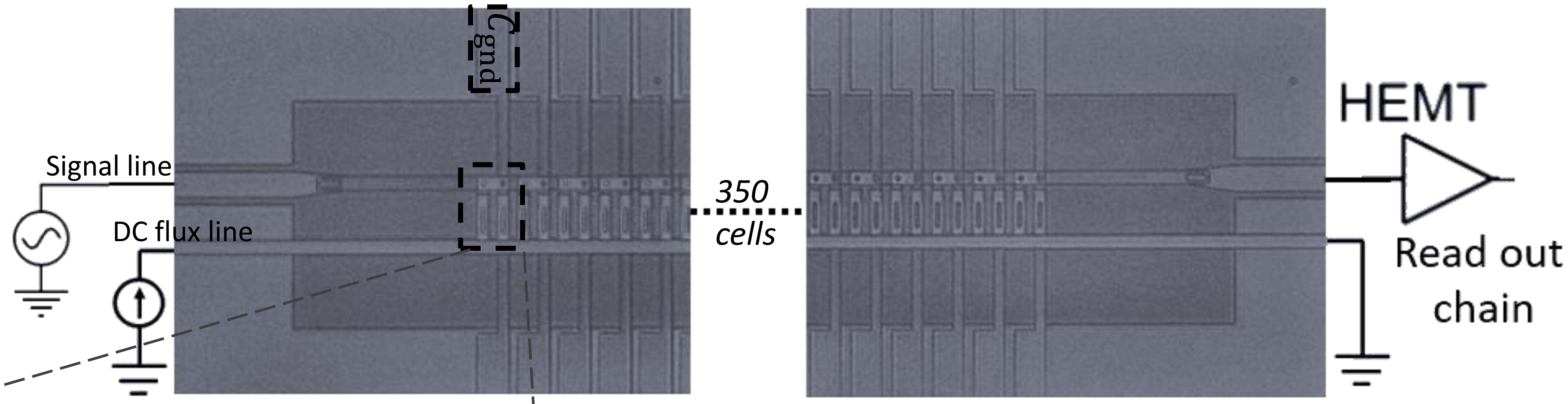
$$g = \sqrt{\left(\frac{k_s^2 k_i^2 (2k_p - k_s)(2k_p - k_i)\omega_p^4}{k_p^6 \omega_i^2 \omega_s^2}\right)\alpha_p^2 - \left(\frac{\kappa}{2}\right)^2}$$

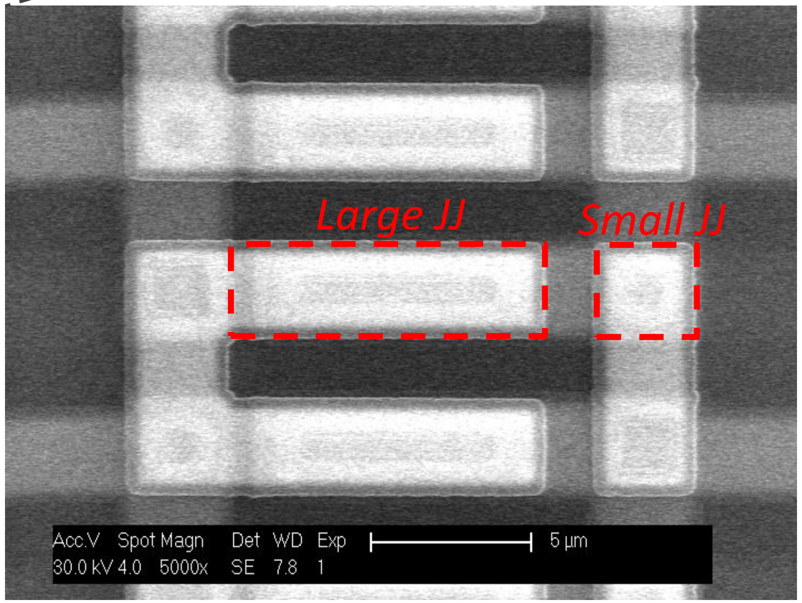
$G_s = |e^{gz}/2|^2$, if $\kappa \approx 0$

<u>Numerical Simulations:</u>

- Large JJ/Small JJ: $r = 6$
- $C_{gnd}$ = 50 fF
- $C_{js}$ = 50 fF
- $C_{jl} = rC_{js}$
- $I_{js0}$ = 1 μA
- $I_{jl0} = rI_{js0}$
- $Z_0 = 50\Omega$

# TWPA design



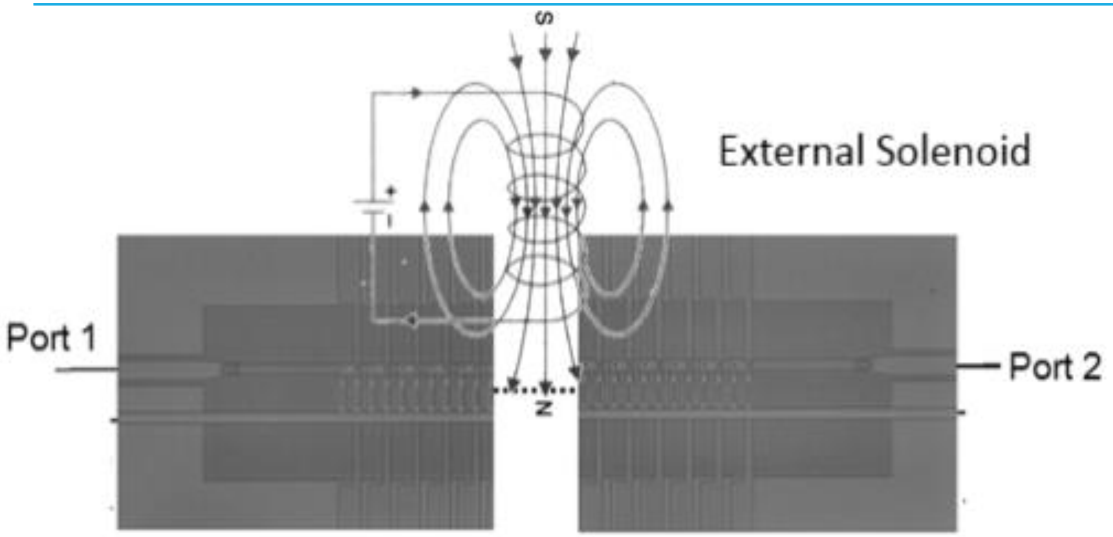



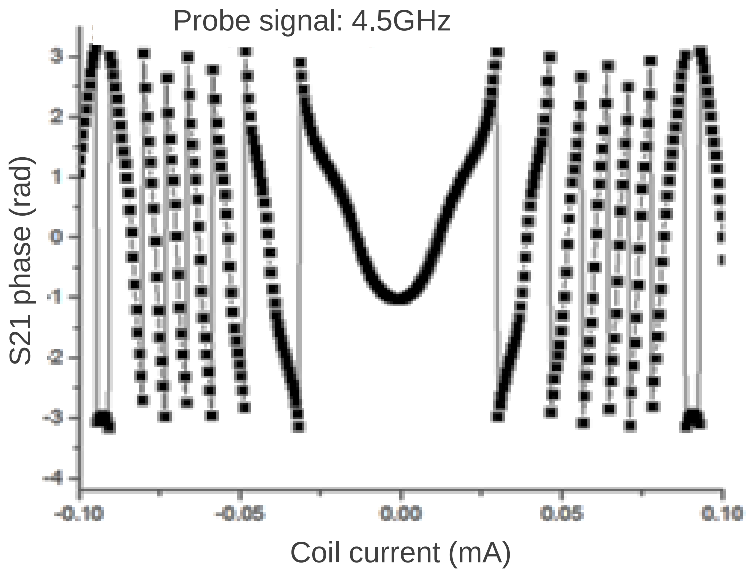

**<u>Designed device parameters:</u>**

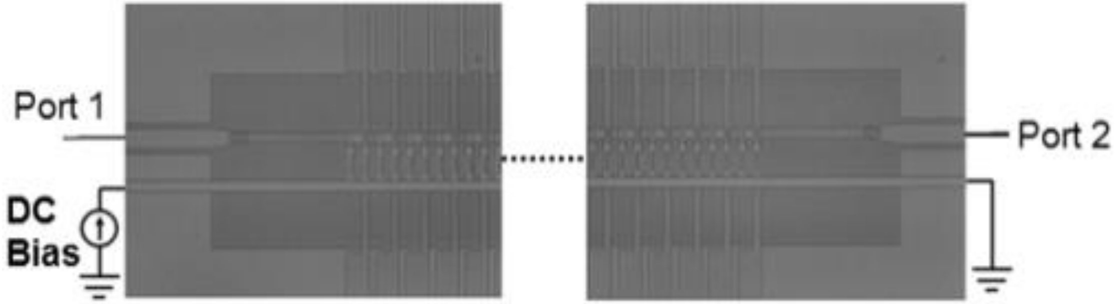
- Small JJs critical current $I_{\text{js}} = 1.2\mu\text{A}$
- Small JJs capacity $C_{\text{js}} = 50\text{fF}$
- Large JJ/small JJ ratio $r = 6$
- Cell capacity to GND $C_{\text{gnd}} = 96\text{fF}$
- $\text{SiO}_2$ dielectric for $C_{\text{gnd}}$
- Length: 350 cells

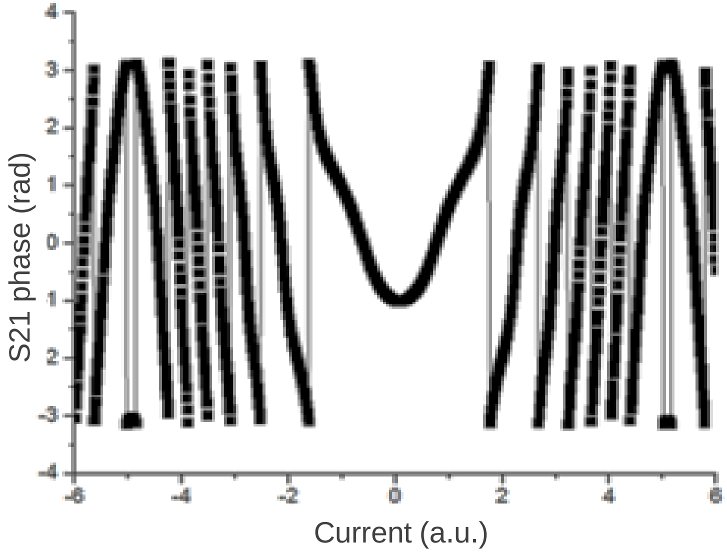
*TWPA fabricated at MIT Lincoln Laboratory (Sergey Tolpygo)*

# Finding field periodicity and full frustration point



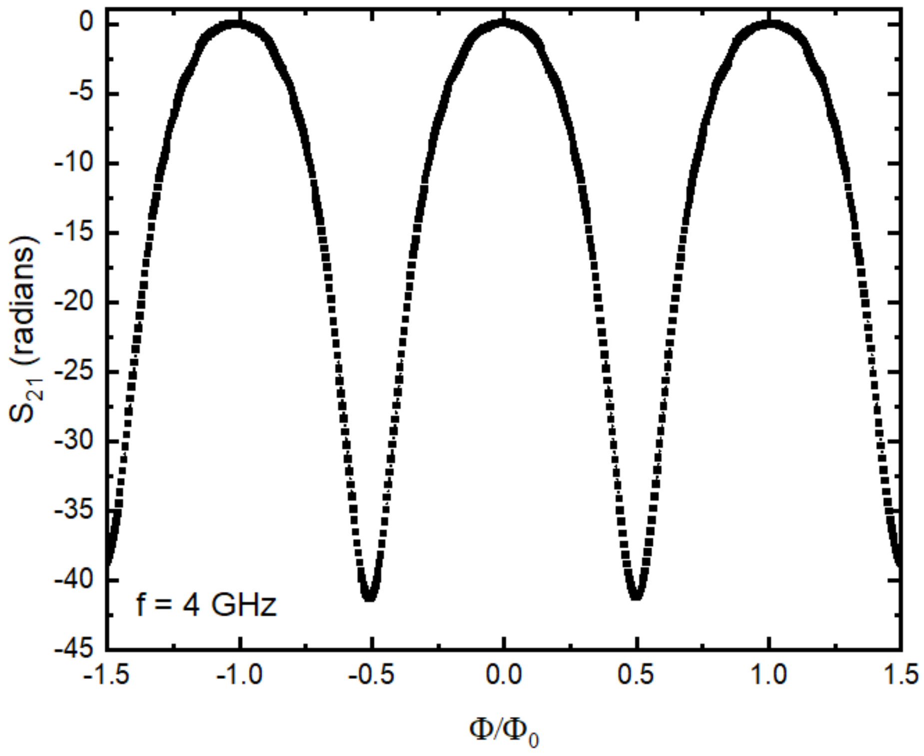



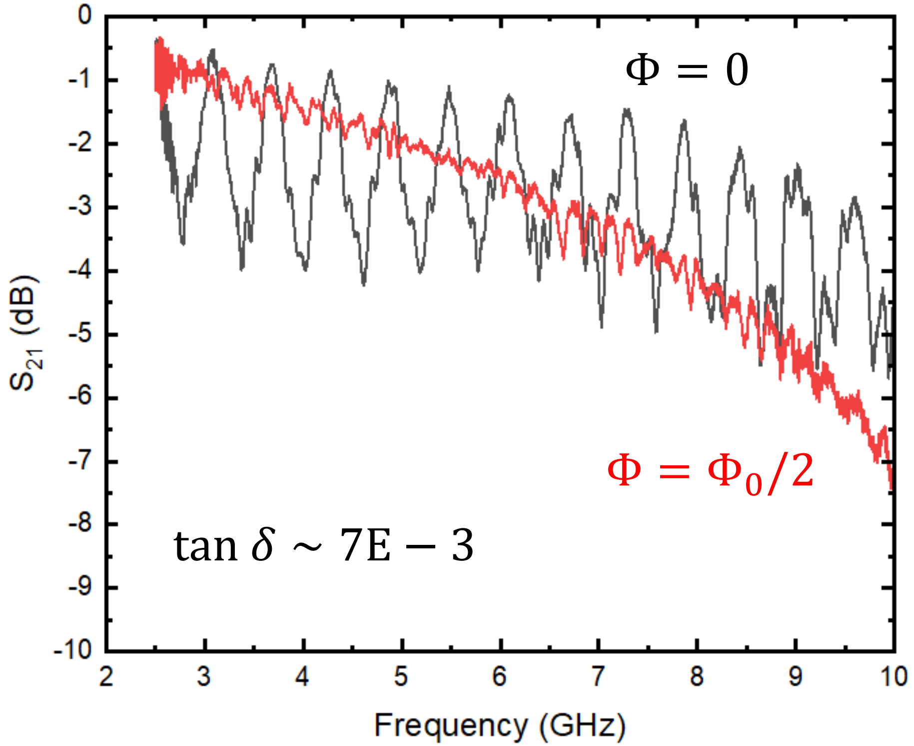




- ➢ Frustration point found by local maximum of phase shift
- ➢ Flux-bias by an external coil or a DC bias line

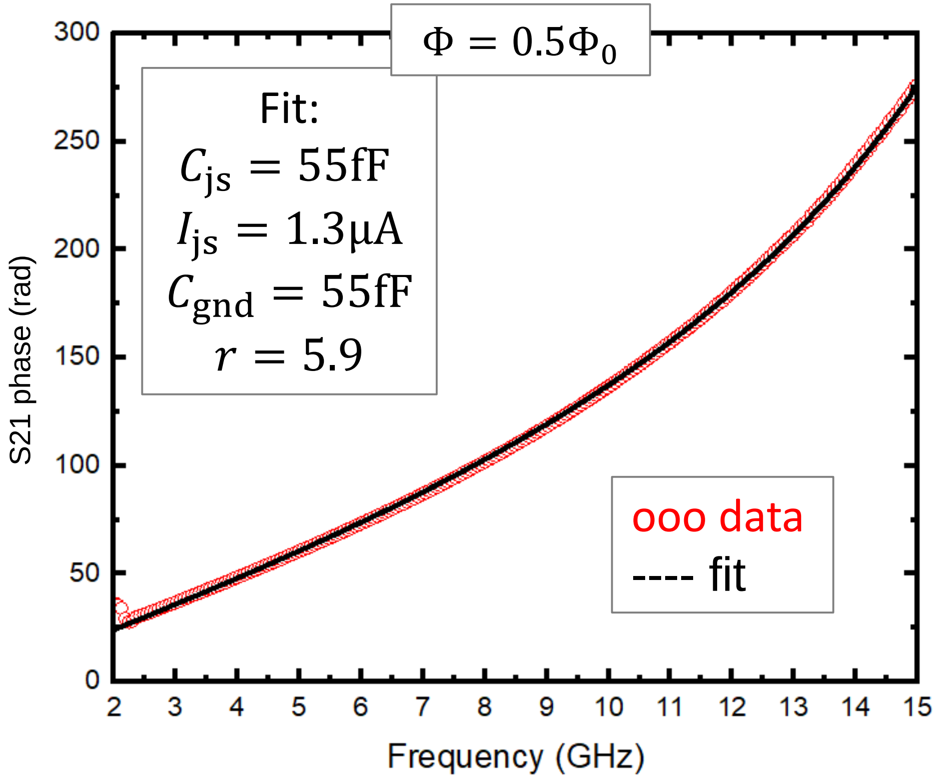
# TWPA insertion loss

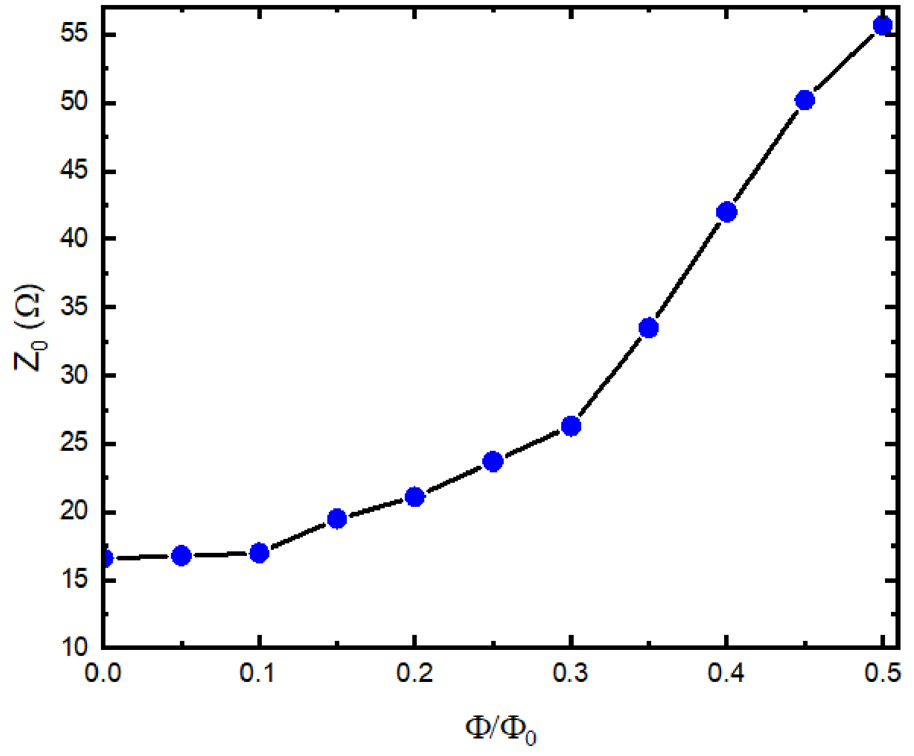



- Low insertion loss: $\sim$ 3dB @ 6GHz , due to short length of TWPA (350 cells, 3mm)
- Flux-tunability of propagation constant and characteristic impedance
- Ripples at $\Phi = 0$ compatible with a mismatched impedance at $Z_0 = 16\Omega$

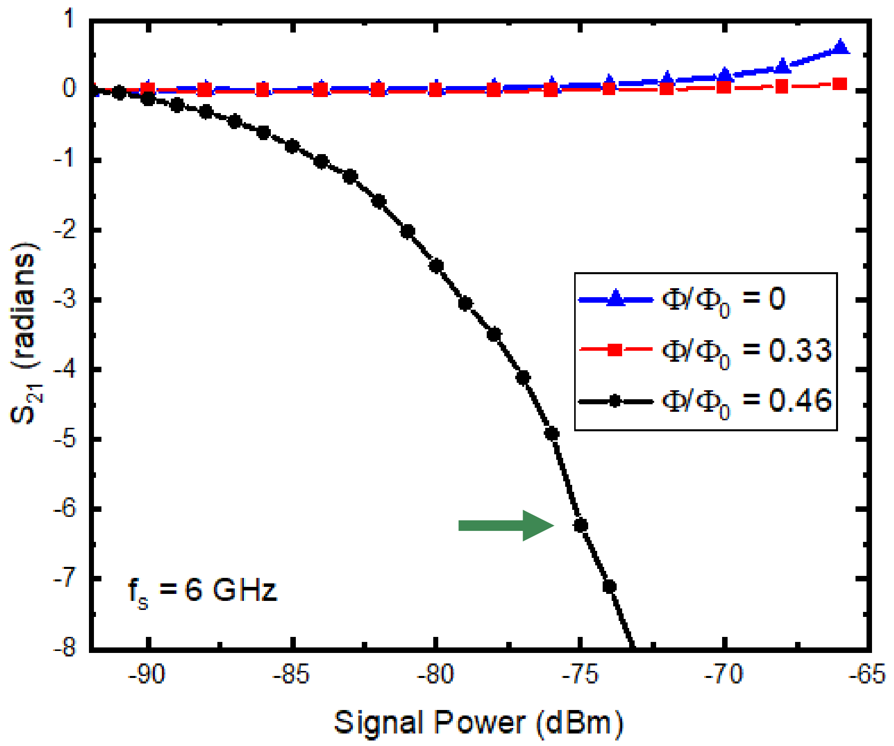
# Propagation constant and characteristic impedance vs flux

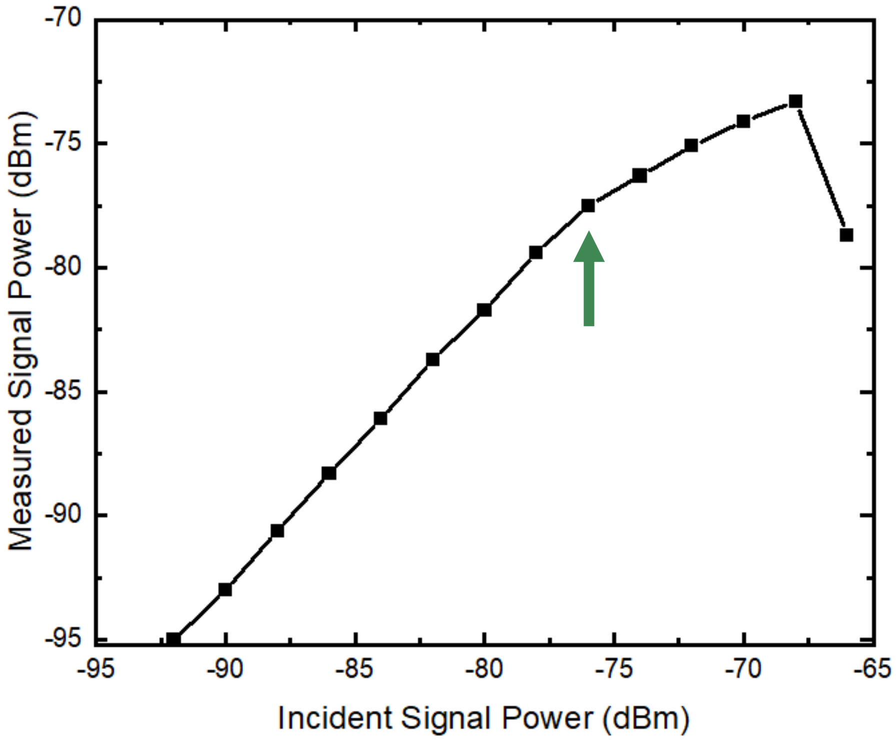



- Fits of $\angle S_{21}(\nu)$ at different values of flux used to estimate $Z_0(\Phi)$. $Z_0$ can be tuned over a broad range for impedance matching
- Fit of $\angle S_{21}(\nu)$ at $0.5\Phi_0$ returns electrical circuit parameters. Fitting simulation by WRSpice
- $C_{gnd}$ independently verified with on-chip resonator measurements

# S21 characteristics vs signal power




- Measured pump self shift to estimate matched gain and mismatched gain:

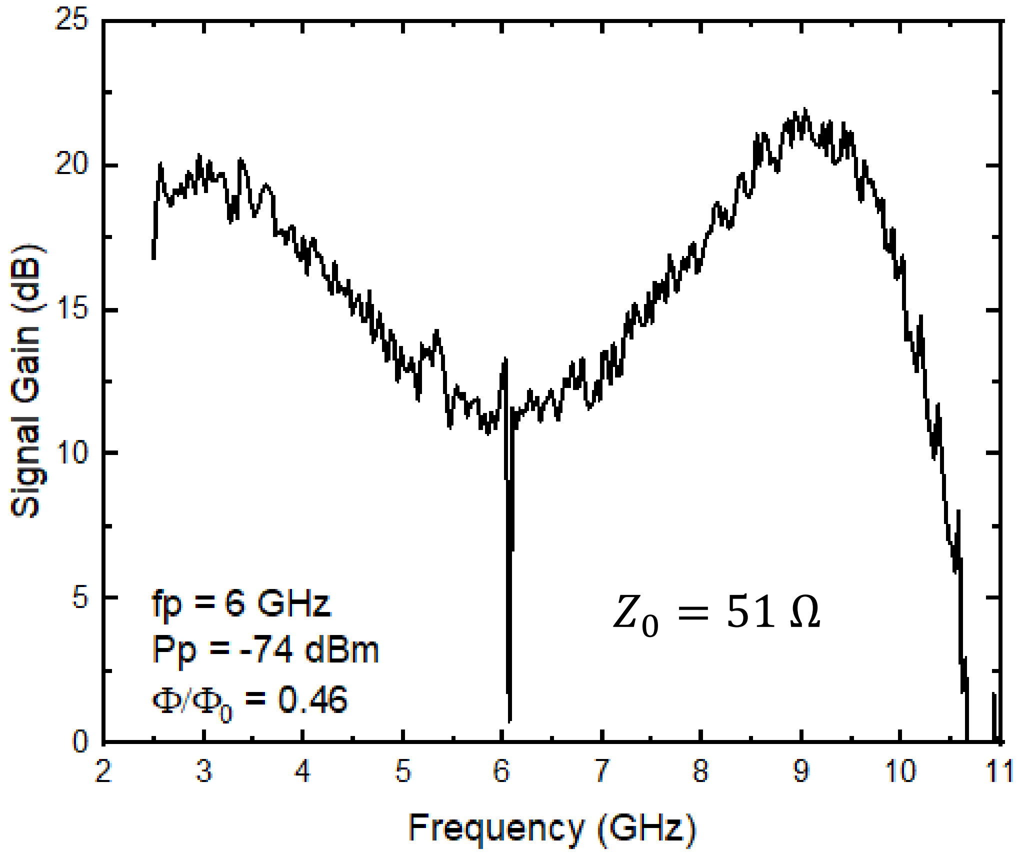
$$G_s = \left| \cosh(gz) - \frac{i\kappa}{2g}\sinh(gz) \right|^2$$

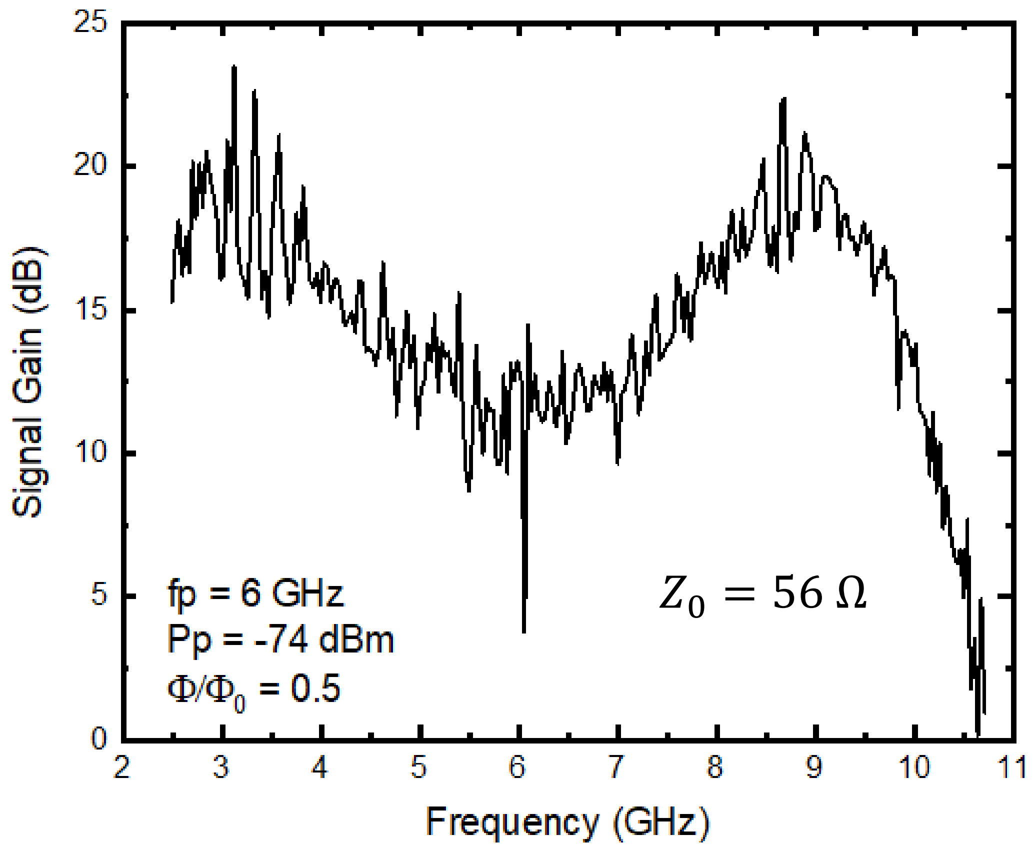
$$G_s = 1 + \phi^2 \sim 11 \text{ dB}$$

$$G_s \approx \frac{exp(2\phi)}{4} \sim 24 \text{ dB}$$

- Kink in and signal power loss due to depletion to higher harmonics

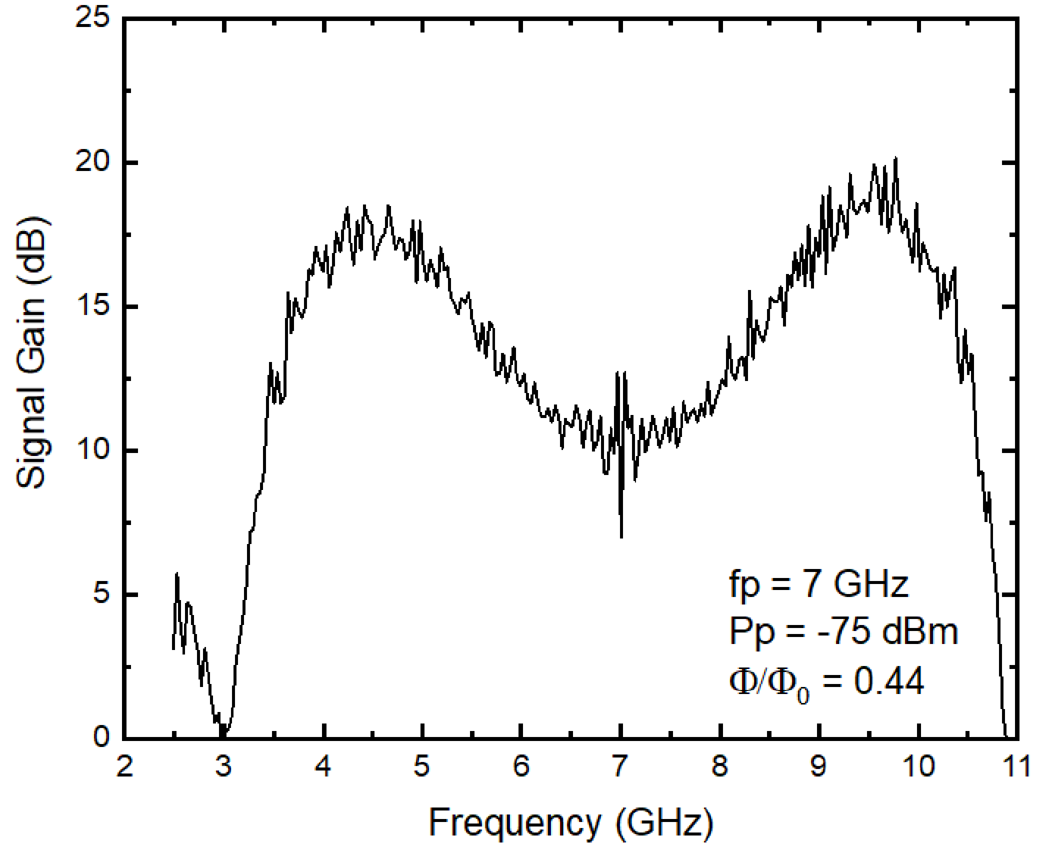
# TWPA broadband gain

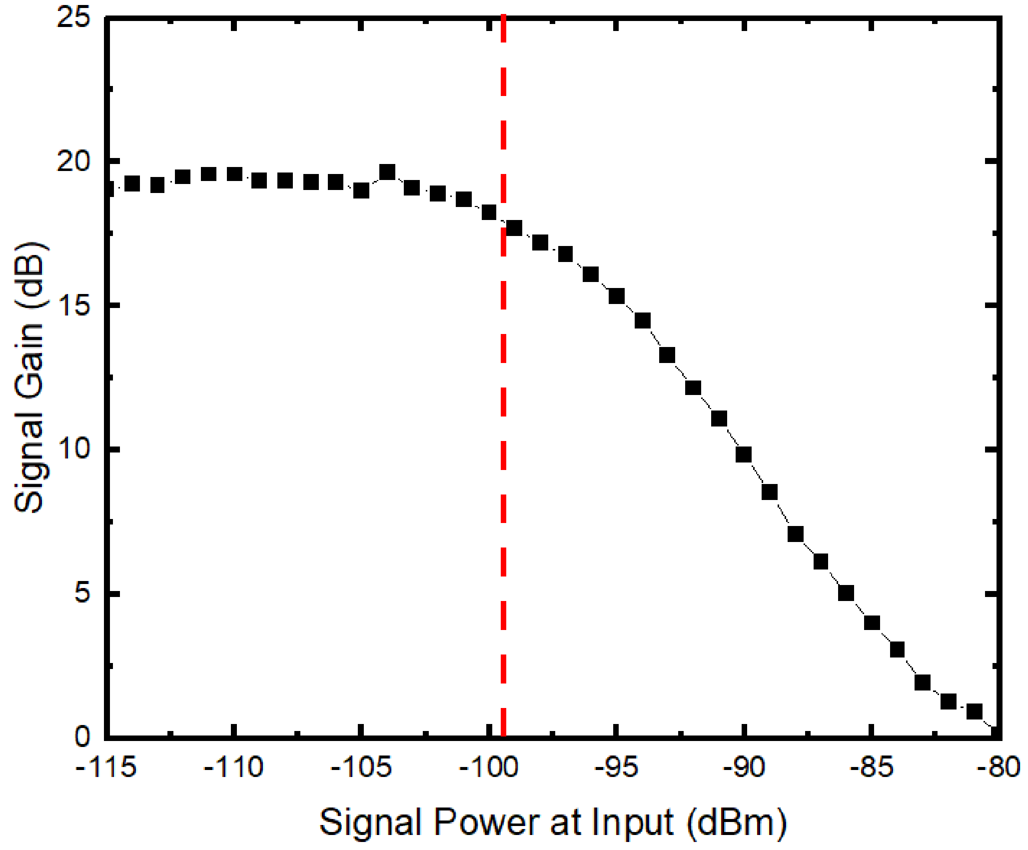



- TWPA able to achieve in excess of 20dB over a short length (3mm)
- Tunability in the characteristic impedance allows for suppression of ripples in gain
- Gain in agreement with estimates of pump self phase shift and measured chromatic dispersion

# TWPA broadband gain




- TWPA pump frequency can be tuned
- 1db-compression point at -99dBm, limited by pump power
- Larger small JJs can increase the 1db-compression point

# Conclusions

- Demonstrated TWPA where phase matching is based on reversing the sign of the Kerr coefficient
- Tuneable gain bandwidth through tuneable pump frequency
- Gain in excess of 20 dB gain
- Large gains over short lengths, implying a low insertion loss
- Tuneable of the characteristic impedance allows for reduction of ripples in gain
- Demonstrated 1 dB compression of -99 dBm

- Currently working on TWPA with silicon nitride dielectric which exhibits insertion loss ~0.5 dB @ 6 GHz and gains in excess of 18 dB